Some Design Considerations for Large Space Structures

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Physical characteristics of large skeletal frameworks for space applications are investigated by analyzing one concept: the tetrahedral truss, which is idealized as a sandwich plate with isotropic faces. Appropriate analytical relations are presented in terms of the truss column element properties, which, for calculations, were taken as slender graphite/epoxy tubes. Column loads, resulting from gravity gradient control and orbital transfer, are found to be small for the class structure investigated. Fundamental frequencies of large truss structures are shown to be an order of magnitude lower than large Earth-based structures. Permissible loads are shown to result in small lateral deflections of the truss due to low strain at Euler buckling of the slender graphite/epoxy truss column elements. Lateral thermal deflections are found to be a fraction of the truss depth using graphite/epoxy columns. A new concept for the truss elements (nestable tapered columns) is shown to require only 25% of the Shuttle cargo bay volume to achieve mass critical payloads. Experimental column buckling and vibration data are presented which verify the predicted structural performance of 5.3 m-long nestable columns. Simplified economic analysis shows that, compared to aluminum, graphite/epoxy nestable columns are cost-effective using Space Shuttle and would be cost-competitive using proposed heavy lift vehicles.

Nomenclature

a	= column spacing ($=\ell\sqrt{3}/2$), Fig. 2
A	= total planform area of truss
A_c	= column cross-sectional area
B	= truss width
D	= plate flexural bending stiffness
d	= column diameter (or nestable column mean diameter)
E	= Young's modulus
E_x, E_y	= laminate Young's modulus in x,y directions
e	= column eccentricity
f	= frequency
G_{xy}	= laminate membrane shear modulus
g_ξ	= a constant defined by Newton's second law [force = (mass × acceleration)/g_ξ, where $g_\xi = 1$ kg·m/N·s^2 (32.2 lbm·ft/lbf·s^2)]
g_0	= acceleration of gravity at Earth's surface
H	= truss depth ($=\ell\sqrt{2/3}$), Fig. 2
I	= moment of inertia
ℓ	= column length
L	= truss length
M	= applied moment
N_x, N_y, N_{xy}	= in-plane stress resultants
P	= axial load in column
P_{crit}	= column buckling load
R	= orbit radius of curvature
R_e	= radius of curvature of Earth's surface
t	= thickness
T	= thrust
W	= truss structural mass
W_c	= column mass
x,y	= rectangular coordinates
α	= coefficient of thermal expansion
α_x, α_y	= laminate coefficient of thermal expansion in x,y directions
ρ	= mass per unit volume
δ	= column lateral deflection
ϵ	= strain
ν	= Poisson's ratio
ν_{xy}	= laminate major Poisson's ratio
θ	= lamination angle
Δ	= truss lateral deflection
η	= thrust-to-mass ratio
λ	= ratio of nonstructural mass to structural mass

Subscripts

c	= column values

Introduction

A NUMBER of future space missions being considered by NASA require ultra-low-mass, large space platforms or structures. Missions such as solar power collection, communications, and Earth resource surveillance are examined in Ref. 1. Although the purposes of these missions vary widely, there appears to be a high degree of structural commonality. A preliminary survey of the anticipated structural requirements is presented in Ref. 2. Some projects require structures that are larger in area than the largest existing Earth-bound structures. The prospect of transporting into orbit structures of this magnitude provides a unique challenge to the aerospace design community to develop extremely efficient structural concepts. The feasibility of these missions will depend on compatibility of the structural concept with available transportation systems and on identifying and developing new, efficient ways to fabricate and/or assemble large structures in space.

The availability of the Space Shuttle in the 1980's will provide a significant increase in man's capability to conduct experiments in space, particularly in those fields requiring large structures. The size of the Shuttle orbiter cargo bay, however, is sufficiently restrictive to render many structural payloads volume-(instead of mass-) limited. Realistically, Space Shuttle will be the vehicle by which any mission involving large structures, or technology development for such a mission, will be accomplished during the next two decades. Thus, it is necessary to consider Shuttle constraints early in any technology development program for large space structures.

The purposes of this paper are to examine the physical characteristics of large space structures subject to constraints that are common to most missions, to estimate some of the

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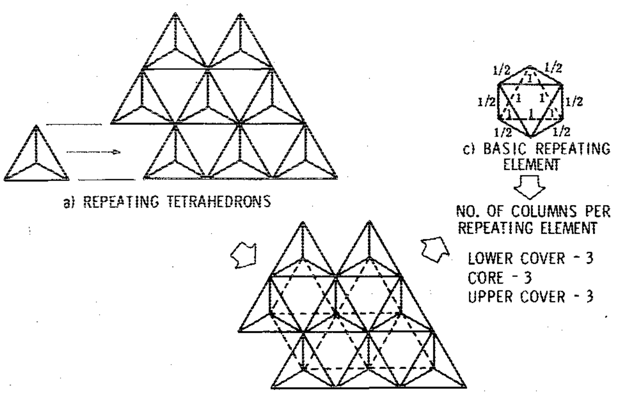

Fig. 1 Tetrahedral truss nomenclature.

loads that may be expected, and to examine the impact of transportation considerations. The paper focuses on one specific representative concept for a large planar structure: the tetrahedral truss (see Fig. 1). Detailed examination of this concept is expected to yield behavioral information characteristic of a large class of low-mass space structures. The tetrahedral truss is idealized as an equivalent sandwich with isotropic faces and a rigid core. Isotropic beam and plate theory are used to determine truss column loads resulting from gravity gradient control and orbital transfer, fundamental vibration frequencies, and deflections resulting from physical and thermal moments. A new concept for a truss element, the nestable column, is presented and shown to eliminate volume-limited payloads using Shuttle for transporting structural components into low Earth orbit (LEO). The impact of transportation on the cost of unassembled components in LEO is examined for both graphite/epoxy and aluminum nestable column elements. Some of the structural advantages of graphite/epoxy over conventional metals are illustrated, and experimental data are given which verify the buckling and vibration characteristics of long, slender graphite/epoxy nestable columns.

Tetrahedral Truss Description

A three-dimensional truss formed by assembling repeating tetrahedrons is shown in Fig. 1. Each tetrahedron is composed of six equal-length columns. The assembly involves connecting the tetrahedrons as indicated in Fig. 1a and subsequently connecting the free apex of each tetrahedron to each of the surrounding apexes with columns of the same length, as shown by the dashed lines in Fig. 1b. A basic repeating planform element of the tetrahedral truss, useful for analysis purposes, is shown in Fig. 1c. The resulting truss structure, when considered from a macroscopic point of view, resembles a sandwich plate composed of identical faces formed by the $(0/\pm 60)$ deg columns and a core formed by the "tripod" columns.

Since the space structures under consideration have very large planform areas, it is reasonable to assume that, for gross behavior, the truss can be idealized as a sandwich plate where the extensional stiffnesses of the faces are averaged over the surface area. It also is assumed that all columns have pinned ends, so that local column bending does not occur. Furthermore, it is assumed that transverse shearing deflections of the truss may be neglected.

The tetrahedral truss herein is assumed to consist of equal length and mass column elements. A more general analysis approach may be found in Ref. 3, where core and facing elements of different lengths and masses are considered. Reference 3 also includes transverse shear and rotary inertia effects. In Ref. 3, it is shown that, when all elements are approximately equal, transverse shearing effects may be neglected without introducing significant error into the

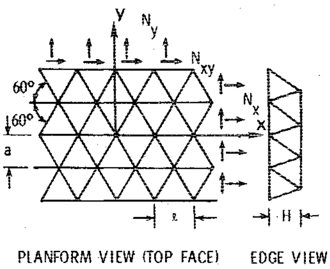

Fig. 2 Tetrahedral truss configuration.

analysis if the characteristic deformation half-wavelength is of the order of 15 to 20 times the plate depth.

The truss coordinate system, dimensional nomenclature, and stress resultants are shown in Fig. 2. A detailed development of the geometric and stiffness relationships and truss analysis is presented in Ref. 4. A summary of pertinent equations from Ref. 4 is presented herein.

Using the previously stated assumptions, the constitutive relations for the sandwich plate covers may be developed. These relations are given by

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \frac{E_c A_c}{a} \begin{bmatrix} 9/8 & 3/8 & 0 \\ 3/8 & 9/8 & 0 \\ 0 & 0 & 3/8 \end{bmatrix} \begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{1}$$

It is seen from the coefficients in Eqs. (1) that each truss cover, formed by the $(0/\pm 60)$ deg columns, is equivalent to an isotropic plate. The extensional stiffness of this equivalent isotropic plate is given by

$$(Et)_{\text{isotropic}} = \frac{E_c A_c}{a} = \frac{2}{\sqrt{3}} \frac{E_c A_c}{\ell} \tag{2}$$

and $\nu = 1/3$. Assuming the covers to be thin isotropic sheets, the bending stiffness of a sandwich plate is given by

$$D = (\sqrt{3}/4) E_c A_c \ell \text{ (plate)} \tag{3}$$

If, however, the truss length is much greater than the width, the truss will behave as a beam, in which case the bending stiffness is given by

$$EI/B = (2/3\sqrt{3}) E_c A_c \ell \text{ (beam)} \tag{4}$$

Considering Fig. 1c, the basic planform repeating element of the tetrahedral truss is seen to contain nine columns, with ⅔ of the columns forming the faces and ⅓ of the columns forming the core. Note that the three core columns are composed of six shared (or ½-column) elements. The structural mass per unit area of this repeating element (and so the entire truss) is given by

$$W/A = 6\sqrt{3}(A_c \rho_c / \ell) = (6\sqrt{3}/\ell^2) W_c \tag{5}$$

By using the appropriate stiffnesses given previously, tetrahedral truss structures may be analyzed with isotropic sandwich beam and plate theory.

Structural Considerations

When analyzing a generic or general class of structures, complete and exact design conditions cannot be specified a priori to aid in selecting the most appropriate material. There are, however, general considerations that can serve as guidelines toward this end.

Material Selection

Large structures for space application are likely to be skeletal frameworks built from long, slender columns. The

Table 1 T300/5208 Gr/E material properties[a]

	Laminate					
	Unidirectional		(90/0_3/90) deg		(90/0_4/90) deg	
Prop.	SI	U.S. cust.	SI	U.S. cust.	SI	U.S. cust.
E_x	131 GPa	19×10^6 psi	99.6 GPa	14.44×10^6 psi	106 GPa	15.35×10^6 psi
E_y	10.9 GPa	1.58×10^6 psi	43.2 GPa	6.26×10^6 psi	36.8 GPa	5.34×10^6 psi
G_{xy}	6.4 GPa	0.93×10^6 psi	6.4 GPa	0.93×10^6 psi	6.4 GPa	0.93×10^6 psi
ν_{xy}	0.32	0.32	0.081	0.081	0.095	0.095
α_x	$-.54\times10^{-6}$/K	$-.3\times10^{-6}$/°F	0.87×10^{-6}/K	0.481×10^{-6}/°F	0.63×10^{-6}/K	0.351×10^{-6}/°F
α_y	29×10^{-6}/K	16×10^{-6}/°F	5.4×10^{-6}/K	3.02×10^{-6}/°F	6.8×10^{-6}/K	3.78×10^{-6}/°F
t[b]	b	b	0.57 mm	0.0225 in.	0.71 mm	0.028 in.

[a] Gr/E density: $\rho_c = 1520$ kg/m^3 (0.055 lbm/in.^{3}). [b] Nominal ply thickness: $t_{0°} = 0.14$ mm (0.0055 in.); $t_{90°} = 0.08$ mm (0.003 in.).

structural behavior of such columns is controlled by overall column (Euler) buckling. Low-mass (efficient) Euler columns require materials with high value of the material stiffness parameter ($E^{1/3}/\rho$). Therefore, one guideline is that the material exhibit a high value of this parameter.

Proposed space structure applications, which include antennae, reflectors, and solar power collection surfaces (or platforms on which such surfaces may be mounted), require not only that a structure undergo small deflections when physically loaded, but also that it be relatively unaffected by thermal loading. Control of thermal distortions may be accomplished several ways, including thermal coatings or insulation. In all cases, however, the thermal distortion problem is eased if the structural material has a very low thermal expansion coefficient, a second guideline requirement. The controlling parameters, material stiffness parameter ($E^{1/3}/\rho$), and thermal expansion coefficient α, are shown in Fig. 3 for various laminates of graphite/epoxy (Gr/E) and for Al, Ti, and steel. The Gr/E material considered herein is T300/5208. The unidirectional lamina properties used, as well as calculated laminate properties, are given in Table 1.

Both ($\pm\theta$) and (0/90) deg laminate families are considered. The abscissa in Fig. 3 is either (θ) deg for the ($\pm\theta$) laminate or (% 90) deg for the (0/90) deg laminate. The dashed curve in the lower plot shows that the ($\pm\theta$) Gr/E laminate exhibits a zero thermal expansion coefficient when (θ) is approximately 42 deg. However, the (± 42) deg laminate has a greatly reduced material stiffness parameter, as shown in the upper plot, being less than the three metals shown. In instances where zero thermal expansion is a definite requirement (e.g., telescope mounts), laminates containing ($\pm\theta$) lamina (such as (± 42) deg) are used to obtain a thermally inert structure. The resultant low modulus is acceptable, since weight is not usually a significant problem for such an application. In large space structures, such an approach is inefficient. However, both the ($\pm\theta$) and (0/90) deg laminates exhibit low expansion and high modulus characteristics at low values of the lamination angle (θ) or a low percentage of (90) deg material.

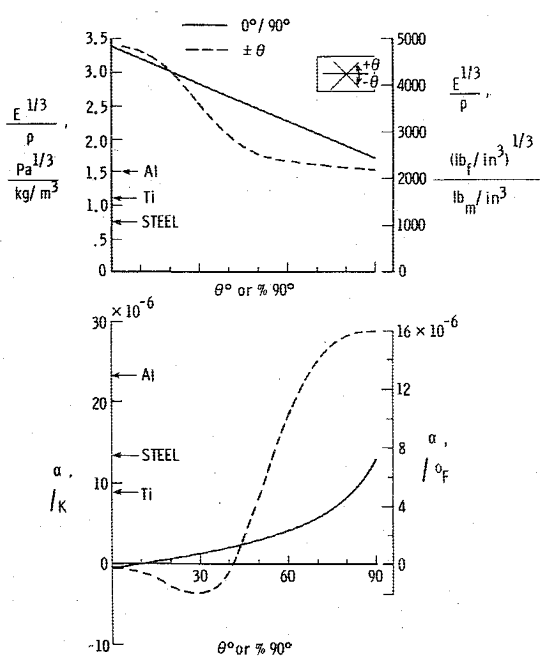

Fig. 3 Material stiffness parameter and thermal expansion characteristics of graphite-epoxy.

Additional transverse characteristics that are not shown are also important. Proper placement of a small percentage of 90 deg material results in a high transverse bending stiffness, which is desirable for local buckling and handling considerations, and a sufficiently low transverse expansion coefficient so that conventional metal tooling can be used for manufacturing. Inasmuch as the (0/90) deg Gr/E laminate family displays significantly higher stiffness and lower thermal expansion characteristics than either aluminum, titanium, or steel and can be tailored to desirable transverse properties, it is used as the basis for study in this paper. The (90/0_4/90) deg laminate, which has a thickness of 0.71 mm (0.028 in.), is used herein for calculation purposes unless otherwise specified. This thickness is somewhat larger than an actual minimum gage but is considered practical for the large-scale structures being investigated. In all cases considered, this thickness is sufficient to preclude local buckling. Overall column buckling is prevented by varying the cylinder radius.

Gravity Gradient Control

Orienting a large truss structure in orbit requires application of control moments or forces. The magnitude of these forces is proportional to the mass of structure being oriented. In turn, the structural mass is determined by the original design load (usually compressive) of each column or truss member. Thus, the individual column load resulting from orbital attitude control (e.g., gravity gradient control) is a function of the column (and truss) mass and may be ex-

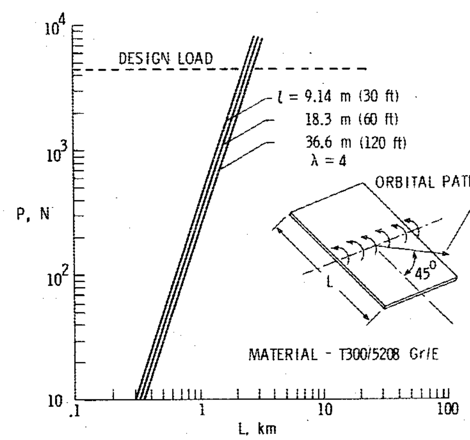

Fig. 4 Column loads due to gravity gradient control moment ($R = 6740$ km at LEO).

pressed as a function of the original column design load. If the induced column load exceeds the original assumed column design load, redesign would be required. The induced column load is given by

$$P=\frac{9}{4}\sqrt{\frac{3}{2}}\left(\frac{\rho_c t_c^{\,1/3}}{E_c^{\,1/3}}\right)R_e^2\left(\frac{L}{R}\right)^3\left(\frac{P_{\text{des}}}{\ell}\right)^{1/3}\left(\frac{g_0}{g_\xi}\right)(1+\lambda) \qquad (6)$$

which was derived[4] assuming the moment distributed along the centerline of a large plate (see Fig. 4), which is required to hold the plate at the most critical inclination (45 deg) to the orbital path. Figure 4 shows the column load resulting from this moment, as a function of the structural span L, for several values of column length ℓ, assuming an original column design load of 4448 N (1000 lbf). A nonstructural mass equal to four times the structural mass also was assumed. It is shown that induced column loads vary between 200 N for $L=1$ km and 4500 N at $L=3$ km. Thus, the original column design load of 4448 N is considered adequate for preliminary design and is used herein to characterize large space structures.

Orbital Transfer

The transfer of a large structural segment from an assembly site in low Earth orbit to a location in geosynchronous orbit requires the application of propulsive force. One method proposed utilizes ion engines, which have low thrust but high specific impulse. It is necessary to determine the column loads that result from those propulsive forces. The following three equations[4] give the compressive column loads that result from accelerating the truss mass [given by Eq. (5)] and any nonstructural mass that may be attached:

$$P=(3\eta/16\sqrt{2})[(W/A)(1+\lambda)L^2]\text{(case 1, Fig. 5)} \qquad (7)$$

$$P=0.022\eta[(W/A)(1+\lambda)L^2]\text{(case 2, Fig. 5)} \qquad (8)$$

$$P=(\sqrt{3}\eta/4)[(W/A)(1+\lambda)\ell L]\text{(case 3, Fig. 5)} \qquad (9)$$

Equation (7) is valid when the thrust is distributed along the truss centerline and applied normal to the plane of the truss (Fig. 5). Equation (8) is valid when the thrust is distributed along two lines equidistant from the truss centerline and applied normal to the plane of the truss. Equation (9) is valid when the thrust is applied along the truss cover edges, in the plane of the truss. Column loads, calculated from these equations, are given in Fig. 5, assuming a column length of 18.3 m (60 ft) and a cylindrical column wall thicknesss of 0.71 mm (0.028 in.). A nonstructural mass of four times the structural weight was assumed ($\lambda=4$). A thrust-to-mass ratio was used ($\eta=0.0098$ N/kg) which results in transfer from low Earth orbit to geosynchronous orbit in approximately one week.[2]. It is shown on the figure that distributing the thrust along a single line (case 1) results in column loads of approximately 4450 N (1000 lbf) for structural segments with spans of 1.6 km (1 mile). Distributing the thrust along two lines (case 2) decreases the resultant column load to approximately 1000 N (200 lbf) for the same span. Furthermore, applying the thrust along the truss over edges results in column loading that is over an order of magnitude less than the original column design load for a span of 1.5 km. These calculations indicate that reasonable concepts exist for accomplishing orbital transfer which do not overload column members.

Natural Frequency of Tetrahedral Truss

The lowest natural frequency (free-free) of a large tetrahedral truss plate with a square planform is given by[4]

$$f_p=\frac{14.1}{4\pi\sqrt{6}}\frac{\ell}{L^2}\sqrt{\frac{E_c g_\xi}{\rho_c(1+\lambda)}}\text{ (square plate)} \qquad (10)$$

If the truss has one dimension significantly larger than the other, it may be assumed to behave as a beam, in which case the lowest natural frequency (free-free) is given by

$$f_b=\frac{22.37}{6\pi\sqrt{3}}\frac{\ell}{L^2}\sqrt{\frac{E_c g_\xi}{\rho_c(1+\lambda)}}\text{ (beam)} \qquad (11)$$

Frequencies calculated from Eq. (10) are shown in Fig. 6 as a function of plate span (L) for several values of column length (ℓ). Nonstructural mass effects are not included. It is shown in Fig. 6 that structures built from 18.3-m columns with a span on the order of 1 km have a lowest natural frequency that is less than 0.1 Hz. This frequency is an order of magnitude lower than encountered for Earth-based strucutres and would be reduced further by the addition of nonstructural mass. The truss plate frequency may be increased by using greater column lengths to reduce the truss mass per unit area. However, practical considerations such as Shuttle transportation or the hard point location requirements of surface attachments effectively limit column lengths that can be used.

Bending of a Tetrahedral Truss

Structures with an ultra-low natural frequency usually are envisioned to be rather flexible and to undergo large deflections. The permissible deflection of such a structure is determined by examining its bending characteristics. The lateral deflection of a plate bent to a cylindrical surface is related to the strain at the extreme fiber by

$$\Delta/H=(\epsilon/4)(L/H)^2 \qquad (12)$$

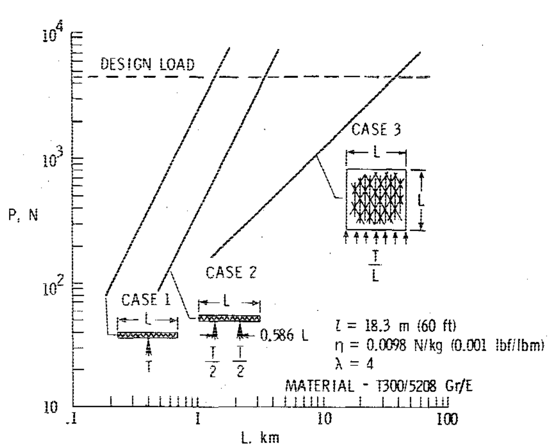

Fig. 5 Colmn loads due to orbital transfer thrust loading.

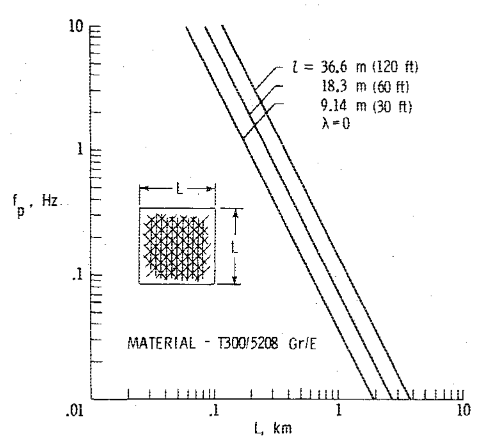

Fig. 6 Fundamental frequency of tetrahedral truss structures ($P_{\text{des}}=4448$ N).

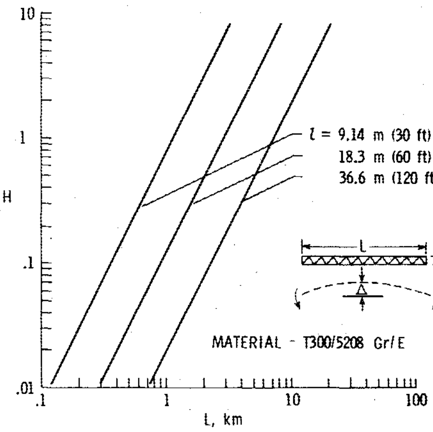

Fig. 7 Deflections of tetrahedral truss structures subjected to maximum allowable bending moment ($P_{des} = 4448$ N).

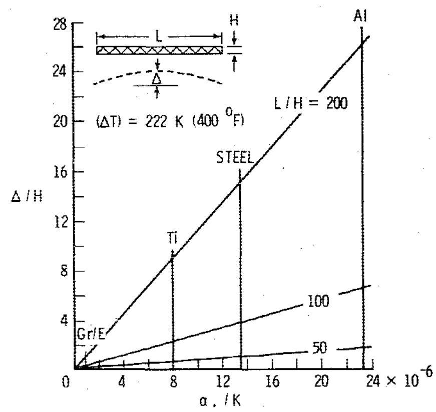

Fig. 8 Tetrahedral truss thermal deflection.

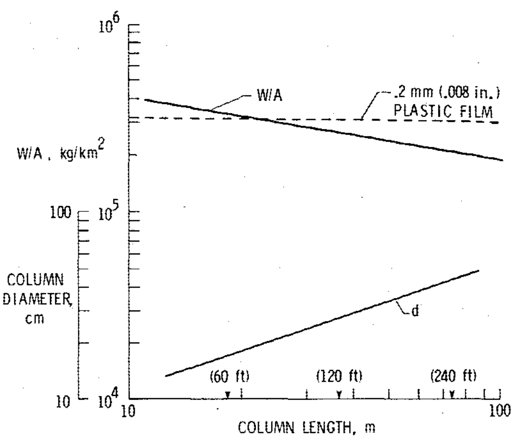

Fig. 9 Tetrahedral truss physical characteristics ($P_{des} = 4448$ N).

This equation was derived[4] solely from geometric considerations. A tetrahedral truss plate would be limited, in bending, to surface strains that are less than or equal to the limiting strain capability of the surface column elements. The critical strain of minimum-gage cylindrical columns that are designed by Euler buckling is given by

$$\epsilon_c = \tfrac{1}{2}\left(P_{crit}/\ell E_c t_c\right)^{2/3} \tag{13}$$

Considering the (90/0$_4$/90) deg laminate and an 18.3-m (60-ft) column length, the strain at buckling of this cylindrical column is found to be 1.093×10^{-4} for $P_{crit} = 4448$ N. This strain is an order of magnitude lower than strains normally encountered in contemporary aerospace structures and indicates that such structures are lightly loaded. The impact of these strains on bending of a tetrahedral truss is illustrated in Fig. 7, which shows the maximum lateral deflection of a truss [see Eq. (12)] bent to a cylindrical surface by a moment that results in surface column loads of 4448 N. Deflections beyond those indicated would result in failure of the surface columns by Euler buckling. It is shown that a truss plate built from 18.3-m columns with a span of 1 km would deflect only approximately 12% of the plate depth. Maximum angular deviation of the surface from a flat condition for this deflection is approximately 0.2 deg. Thus, allowable control forces or moments (which do not buckle the surface columns) will not result in bending deflections that are significant for many applications such as solar energy collection surfaces, reflectors, or concentrators.

Truss Thermal Deflection

The lateral deflection of a truss due to a temperature difference between the two faces is given by[4]

$$\Delta/H = (\alpha_c \Delta T/8)(L/H)^2 \tag{14}$$

where ΔT is the temperature difference between truss surfaces. Deflections calculated assuming a temperature difference of 222 K (an assumed worst case) between the two truss faces are presented in Fig. 8 as a function of the truss thermal expansion coefficient, which, in this case, is equal to the column value. Three values of truss span-to-depth ratio are shown. The vertical bars identify individual material expansion coefficients. For a span-to-depth ratio of 100 ($L/H = 100$), the thermal deflections of a Gr/E truss are the same order of magnitude as the maximum permissible mechanical deflections: a fraction of the depth. Titanium, steel, and aluminum truss deflections are shown to be significantly higher.

Truss Physical Characteristics

The preceding sections established various aspects of the structural behavior of large skeletal frameworks, built from thin-walled Gr/E cylindrical colums. It is enlightening to examine some physical characteristics of such structures, to obtain a perspective on the nature of minimum-mass, large space trusses relative to more familiar references. The diameter of columns designed for a 4448-N compressive load, and resultant mass per unit area of a tetrahedral truss built from these columns, is shown in Fig. 9 as a function of column length. The mass per unit area of 0.2-mm (8-mil) plastic film also is shown as a dashed line. It is seen that a truss structure, built from 18.3-m-long (60-ft) columns will have a mass per unit area which is equivalent to this plastic film. Columns 18.3 m long are shown to have a diameter of 17 cm (6.7 in.). Thus, a truss structure built from thin-walled tubes, which are approximately three times as long and only slightly smaller in diameter than telephone poles, has a mass per unit area which is roughly equivalent to a thin plastic sheet. It is recognized easily that surface coverings such as solar collectors will have mass requirements that far exceed that required by the supporting structure.

Previous results, illustrated in Figs. 4 and 5, showed that an assumed original column design load of 4448 N was sufficient to accomplish gravity gradient control and orbital transfer, including the effects of nonstructural mass, for trusses with spans of 1 to 3 km. It also was shown in Fig. 7 that the maximum lateral deflections are small when a truss is subjected to a bending moment such as that required for gravity gradient control. For the columns discussed previously and a span of 1 km, the truss ratio of lateral deflection to span (Δ/L) is found to be approximately 1.8×10^{-3}, which is the same order of magnitude as required by municipal building codes to prevent plaster ceilings from cracking. Thus, large space trusses will move as relatively rigid bodies when subjected to control moments or forces in orbit.

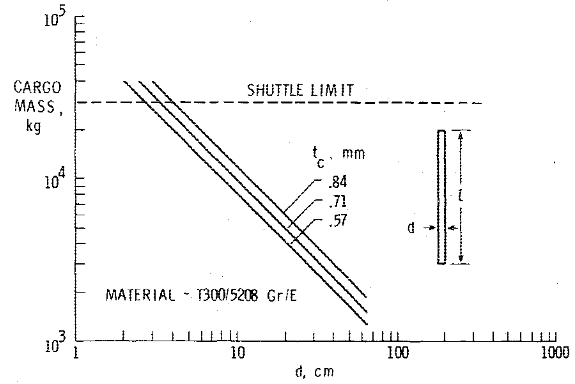

Fig. 10 Packaging characteristics of tubular columns.

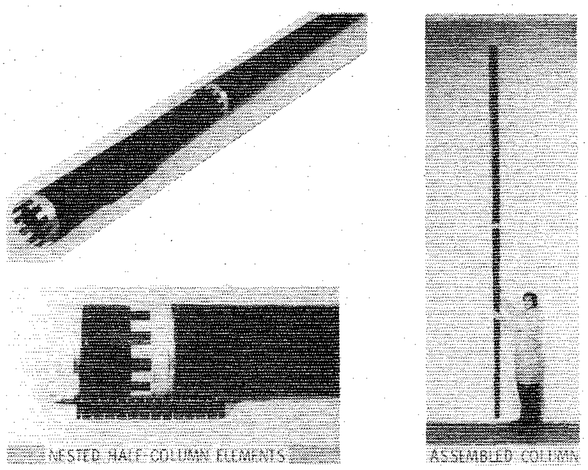

Fig. 11 Photographs of Gr/E nestable column (ℓ= 5.2 m).

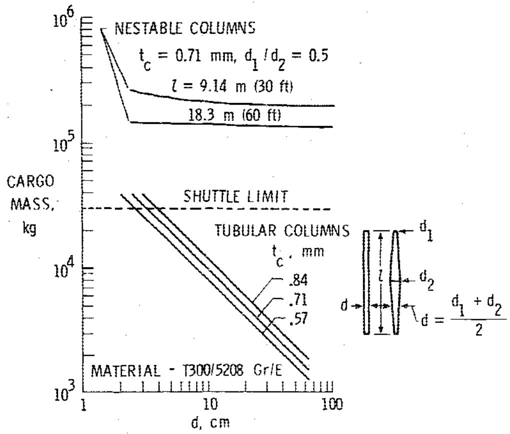

Fig. 12 Comparison of nestable and tubular column packaging characteristics.

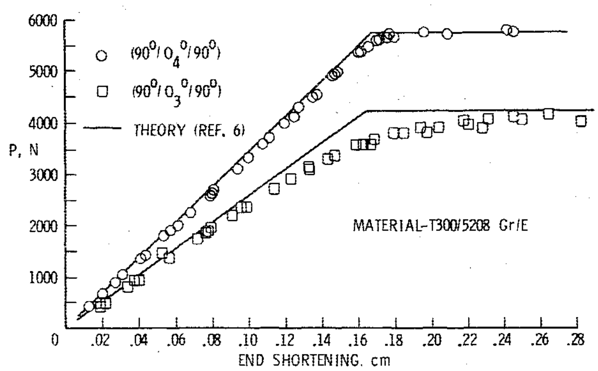

Fig. 13 Experimental load, end-shortening response of nestable columns.

Transportation Considerations

The physical characteristics of large tetrahedral trusses just discussed were obtained considering the column members to be thin-walled cylindrical compression elements due to the high structural efficiency of such configurations. It is very difficult, however, to transport such members efficiently. The ratio of displaced volume to the mass of an efficient cylindrical column is very high. Thus, a large volume is required to transport a small mass of identical, thin-walled cylindrical tubes. The maximum cargo mass obtainable when using Space Shuttle for transporting thin-walled cylindrical columns is shown in Fig. 10. It is shown on the figure that mass critical payloads can be achieved only with very small tubing (e.g., 3 cm diam). Payloads with column diameters in the range of interest are highly volume-limited (e.g., $\leqslant$30,000 kg), which is extremely inefficient from a transportation viewpoint.

Another concept for the column members of large space structures is presented in Ref. 5. This concept, shown in Fig. 11, uses tapered half-column elements that nest like plastic cups, for transportation into orbit. On orbit, these elements are assembled by joining together the large ends of two half-columns. The aluminum center joint shown in Fig. 11 is composed of two identical halves and requires only axial motion and force to lock. The resultant column, with a taper ratio of one-half, will carry approximately 30% more load before buckling than an equal-mass, constant-diameter cylinder. Additionally, a tapered column has approximately 10% less mass than a constant-diameter column that will carry the same load.

The maximum cargo mass in the form of nestable columns which can be transported by Space shuttle, and the resultant advantage over constant diameter cylinders, is shown in Fig. 12. The nestable column curves, calculated from equations presented in Ref. 5, indicate that mass critical payloads are obtained easily using efficient column designs. In fact, considering 9.14-m half-column elements (18.3-m-long columns), it can be shown that 30,000 kg of nestable columns can be stacked into approximately 25% of the cargo bay volume, which could be advantageous in overcoming Shuttle payload center-of-gravity limitations.

Although not shown, multiple (e.g., quadruple, etc.) segment, nestable columns may be constructed and efficiently transportated which have lengths that are many times the Shuttle cargo bay length. The nestable concept also may be applied to tapered open "lattice" columns, provided that the structural efficiency and economics of using such truss elements can be justified.

Nestable Column Experiments

A program currently is under way at the NASA Langley Research Center to obtain data on the structural behavior of long, slender, thin-walled columns for use in constructing large structures in space. Efforts to date include theoretical and experimental studies to verify primary failure modes and vibration characteristics of long slender columns. Since the column elements of large trusses will operate at extremely small strains, it was of interest to determine if material properties calculated from larger strains were adequate for predicting structural behavior in this regime. Columns approximately 5.2 m (17 ft) long with both $(90/0_3/90)$ deg and $(90/0_4/90)$ deg laminate walls (see Table 1) have been fabricated and tested. The center diameter of the columns was 10.2 cm (4 in.), and the end diameter was 5.1 cm (2 in.). The columns were simply supported and tested vertically in compression. The load, end-shortening response of these nestable columns is shown in Fig. 13. It is shown in the figure that the columns closely followed the theoretical predictions[6] for overall column buckling. Since overall buckling of long slender columns occurs at very low strain values, postbuckling material damage does not occur, and the buckling data were

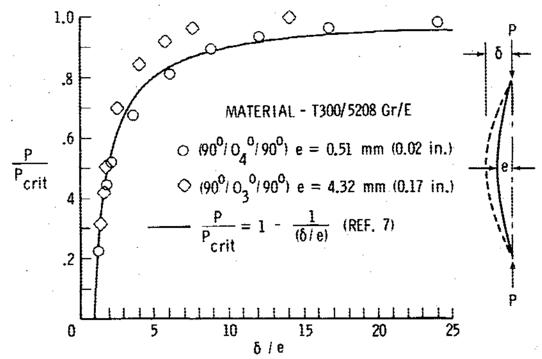

Fig. 14 Experimental load, lateral deflection response of nestable columns.

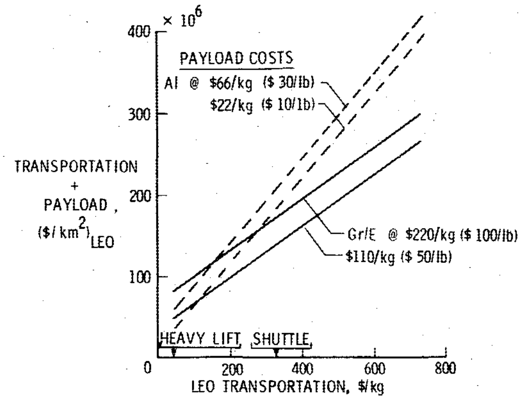

Fig. 15 Costs per unit area of unassembled tetrahedral truss structure in low Earth orbit using nestable columns (ℓ = 18.3 m).

obtained (and found repeatable) from a number of tests of a single specimen. Theoretical axial stiffnesses were verified using local strain and end-shortening measurements. Initial column eccentricities were measured, and lateral deflections were monitored during stability testing. These experimental results are presented in Fig. 14, where it is shown that the experimental behavior generally follows the theoretical[7] lateral deflection predictions of a uniform-cross-section column. Based on results to date, the stability of nestable columns is found to be governed by overall buckling and is predicted closely by strength of materials techniques, which include the effects of taper.

The dynamic behavior of these columns was examined by conducting both free-free and simply supported vibration column experiments. The theoretical and experimental results are given in Table 2. As with the stability results, the column natural frequencies were predicted closely. A shell-of-revolution computer program,[8] which accurately accounts for transverse shear and rotary inertia effects, was used to generate the theoretical values of Table 2.

Cost Considerations

The results presented thus far have illustrated the technical advantages of using Gr/E for fabrication of large space structures as opposed to using a more conventional, less costly material such as aluminum. In order to determine whether aluminum or Gr/E is more economical, first consider the following equation, which gives, in structural efficiency form, the mass of an optimum cylindrical column designed by Euler buckling in the minimum-gage regime:

$$W_c/\ell^3 = (2\rho_c/E_c^{1/3})(t_c/\ell)^{2/3}(P/\ell^2)^{1/3} \quad (15)$$

This equation may be written for aluminum and Gr/E columns of the same length and design load. The mass ratio $(W_{Al})/(W_{Gr/E})$ then may be formed. This ratio is given by

$$W_{Al}/W_{Gr/E} = (\rho_a/\rho_g)(t_a/t_g)^{2/3}(E_g/E_a)^{1/3} \quad (16)$$

where subscripts a and g refer to Al and Gr/E, respectively. It can be shown that this ratio is valid for nestable, as well as cylindrical, columns. Using an aluminum thickness of 0.51 mm (0.020 in.) and a Gr/E thickness of 0.71 mm (0.028 in.) with the appropriate moduli and density (see Table 1) results in $W_{Al}/W_{Gr/E} = 1.69$. Consequently, assuming mass critical payloads, 69% more Shuttle flights would be required to orbit a given area of aluminum structure than would be required for Gr/E structure.

The cost per unit area of unassembled tetrahedral truss structure in low Earth orbit, as a function of transportation and fabricated structural element costs, is given by

$$(\$/A)_{LEO} = [(\$/W)_{transportation} + (\$/W)_{structure}](W/A) \quad (17)$$

where W/A is given by Eq. (5). A plot of Eq. (17) is shown in Fig. 15 for a truss structure built from 18.3-m-long columns. Anticipated Gr/E and Al structure costs are bracketed by the estimated costs per kilogram shown in Fig. 15. It is seen that, at or above current Shuttle transportation costs (e.g., \$10 million/flight ≃ \$340/kg), it is clearly cost-effective to use the more expensive Gr/E structure, since the lower costs of aluminum structure are more than offset by the increased transportation requirements. Furthermore, the results shown in Fig. 15 indicate that Gr/E structure may be cost-competitive even if a heavy lift vehicle (or advanced Shuttle) is developed. Since the mass per unit area (W/A) parameter in Eq. (17) is shown in Fig. 9 to decrease with column length, the cost per unit area $(\$/A)_{LEO}$ also will decrease with increased column lengths.

Table 2 Vibration results for nestable columns

	Free-free		Simple-support	
Laminate	Theo, Hz	Exp, Hz	Theo, Hz	Exp, Hz
$(90/0_3/90)$ deg	32.0	30.3	12.0	12.6
$(90/0_4/90)$ deg	33.9	...	12.7	13.2

Concluding Remarks

Physical characteristics of large space structures are investigated by analyzing one concept: the tetrahedral truss. This structure is shown to be equivalent, macroscopically, to a sandwich plate with isotropic faces. The appropriate stiffness relations and analytical equations that permit investigating the truss structure as an equivalent isotropic plate are presented in terms of the truss column element properties.

It is shown that large, efficient space structures of the class investigated herein will be lightly loaded. Since all physical applied loads result from inertia loading, reduction of structural mass results in lower induced element loads. Use of the most efficient material (Gr/E) to accomplish mass reduction appears warranted. Although light loads are anticipated during on-orbit operations, it is likely that the design load for the truss column elements may be dictated by other considerations. For example, loads resulting from on-orbit assembly could be the principal design consideration. Alternately, it may be that truss stiffness for maintaining acceptable vibrational behavior is the major concern, since the fundamental frequencies of large truss structures are shown to be at least an order of magnitude lower than large, contemporary Earth-bound structures. Efficient Gr/E truss structures are found to display a mass per unit area equivalent to thin (0.2-mm) plastic film. The low strain at buckling, which characterizes efficient truss column elements, is shown to limit transverse deflections of the structure. Permissible loads will result in lateral deflections that are only a fraction of the truss depth. Similarly, out-of-plane thermal deflections cause by a temperature gradient through the truss depth are reduced greatly using Gr/E columns having a low thermal expansion coefficient.

A concept for the truss elements, nestable tapered columns, is shown to require only approximately 25% of the Shuttle cargo bay volume to achieve mass critical payloads. Additionally, the nestable columns are structurally more efficient than minimum-mass, constant-diameter cylinders. Experimental column buckling and vibration data are presented which demonstrate that Gr/E nestable columns are efficient, predictable, and reliable structural elements.

Simplified economic considerations show that the minimum cost of space structure in orbit (unassembled) is highly transportation-dominated. It is shown that reduced structural mass requirements associated with nestable Gr/E columns results in sufficient savings in transportation costs so that the higher-priced Gr/E structure is cost-effective using the present Space Shuttle transportation system. Furthermore, Gr/E structure would be cost-competitive using proposed advanced Shuttle or heavy lift launch vehicles.

References

[1] "Benefit/Cost Study of Large Area Space Structures," NASA CR-144936, 1975.

[2] Hedgepeth, J. M., "Survey of Future Requirements for Large Space Structures," NASA CR-2621, 1975.

[3] Noor, A. K., Greene, W. L., and Anderson, M. S., "Continuum Models for Static and Dynamic Analysis of Repetitive Lattices," *Proceedings of the AIAA/ASME 18th Structures, Structural Dynamics and Materials Conference*, San Diego, Calif., March 1977.

[4] Mikulas, M. M. Jr., Bush, H. G., and Card, M. F., "Structural Stiffness, Strength and Dynamic Characteristics of Large Tetrahedral Space Truss Structures," NASA TM X-74001, 1977.

[5] Bush, H. G. and Mikulas, M. M. Jr., "A Nestable Tapered Column Concept for Large Space Structures," NASA TM X-73927, 1976.

[6] Timoshenko, S. P. and Gere, J. M., *Theory of Elastic Stability*, Mc-Graw-Hill, New York, 1961.

[7] Shanley, F. R., *Strength of Materials*, McGraw-Hill, New York, 1957.

[8] Cohen, G. A., "User Document for Computer Programs for Ring-Stiffened Shells of Revolution," NASA CR-2086, 1973.